

# Marginal Value Approach to Pricing Temperature Options and Its Empirical Example of Daily Temperatures in Nagoya

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## Abstract

This paper focuses on temperature options pricing using the marginal substitution value approach proposed by Davis. We first extend a utility function dealt with in Davis's work to the functions of "HARA" type which are well-known as a general class of utility functions in theoretical economics. The extension allows us to analyse the sensitivity of "temperature options" price to a risk aversion coefficient. In this framework, we next evaluate temperature option price contract on accumulated cooling degree days (CDDs) for an electric power generator. As an illustrative example, a time series model of daily air temperature at Nagoya city in Japan is estimated and on the basis of the model and our pricing formula, the temperature option prices for CDDs in summer are calculated numerically. The result verifies that the temperature put option has the property of an insurance product for the purchaser.

## 1 Introduction

Weather derivatives are financial products whose underlying assets are given by some changes in weather conditions. Their history dates back to 1997, when Enron and Koch transacted first degree-day swap in the winter. From then on, against a background of intense competition and destabilization in business environment of companies due to global liberalization and deregulation, weather derivatives have been mainly developing as a risk-reduction tool for companies which are exposed to weather risk.

"Temperature options" are treated in the present article mainly as typical products of

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weather derivatives, whose underlying asset are some temperature indices. It is well-known that the “accumulated heating/cooling degree days (HDDs/CDDs)” are commonly used as such underlying assets. These are calculated by accumulation of differences between a fixed basis temperature and the actual temperatures for a certain fixed period. Since these indices highly correlate with profit of some companies, especially energy companies supplying electric power or gas, temperature options on HDDs and CDDs are transacted for risk reduction strategy by such companies.

However, one can not simply apply the “risk-neutral pricing method” (see Nielsen [8]) in a complete market as typified by Black and Scholes, to pricing of temperature options, since temperature movement is not a tradable asset as mentioned above. Therefore, for this pricing problem, some attempts on the basis of the expected utility theory have been reported (e.g. Cao and Wei [1]). The approach proposed by Davis in 2001 [3] is also one of such proposals included in this framework. This approach may be summarised as follows. First, temperature movement is simulated by an AR(1) model. Underlying assets HDDs or CDDs, which are defined as accumulations of temperature index given by the temperature model, are described as a “geometric Brownian motion process” which is familiar in physics and financial engineering. Next, by using the marginal substitution value approach in the expected utility theory, an explicit and simple pricing formula of the temperature derivatives in the above model for the investor with a logarithmic CRRA (Constant Relative Risk Aversion) utility is derived.

Thus, in Davis’s work, a purchaser has a logarithmic CRRA utility. On the other hand, a utility function which is applied to pricing often belongs to a wider class of such functions. Moreover, in order to analyse the degree of risk aversion of a purchaser of temperature options, we must extend the class of utility functions dealt with in Davis’s work to a more general one. Such an analysis is important, since temperature options may be regarded as a sort of insurance product. Indeed, daily temperature data is only a physical index and is not tradable in market, whereas the insurance value of the derivatives depends on the trading player. Moreover, for the company whose future profit correlates with temperature, the value is enormous. Consequently, the purchase of a temperature option needs to take account of a hedgeable temperature risk value in pricing them.

In consideration of the facts mentioned above, it seems natural that we generalize a class of utility functions in the framework of Davis’s work and carry out a risk analysis for the purchaser; this is the main purpose of the present article. That is, we will formally derive a

general pricing formula of temperature derivatives by the extension of the class of the utility function dealt with in Davis's work to "HARA (Hyperbolic Absolute Risk Aversion)" functions, because the HARA class is a general class of the utility functions with a parameter corresponding to investor's risk aversion and hence we may investigate the sensitivity of temperature options price with respect to the risk aversion parameter.

This paper is organised as follows. In Section 2, we first review Davis's framework and give some notations for HDDs/CDDs and options on them. In his framework, an option pricing formula for energy companies with logarithmic CRRA utility function is given, which is similar to the Black-Scholes formula for call-options in a complete financial market model. Under the same assumptions as those in Davis's work, we will next relax an assumption on the utility function and extend the option pricing formula to that for utility functions of HARA class. Then, because it is difficult to derive the explicit formula in the case of the general utility functions belonging to the HARA class, we derive the pricing formula in a form suitable for calculating the approximate value numerically with Monte Carlo simulation.

In Sections 3 and 4, by using the new formula together with computer simulation, we give an empirical example of pricing call and put temperature options on CDDs in an analogous way to that in Davis's paper. As the actual temperature data, then, the daily temperatures at Nagoya city are adopted. Briefly, Nagoya has a large population of almost two million and the city is geographically located roughly in the center of Japan which is regarded as the major urban and one of the foremost industrial areas. For instance, the headquarters and production plants of Toyota Corp. are located close to Nagoya. Thus, Nagoya is a typical large city in Japan, and this is the reason why we use its temperature data as an example. In Section 3, we estimate a time series model of the daily air temperatures at Nagoya. Moreover, through this model we identify the geometric Brownian motion describing the underlying assets CDDs and examine the adequacy of the model. In Section 4, as an illustrative example, we calculate call and put option prices on the CDDs in summer for an electric power company with HARA utility in terms of our pricing formula and Nagoya temperature model. Then, we investigate the risk sensitivity of temperature options with respect to a risk aversion parameter which is contained in the utility functions of HARA class. In particular, the results on the put option will indicate that such an option has the property of an insurance product. Finally, in Section 5, we give some concluding remarks.

## 2 A Temperature Option Pricing Model and the Option Pricing Formula

In this section, we first review the pricing model of temperature derivatives proposed by Davis and his pricing formula, and then proceed to an extension of his framework.

We start with some definitions of weather indices. Let  $T_t$  be the average of the temperatures in degrees on date  $t$ . The daily number of “heating degree” HDD and “cooling degree” CDD on date  $t$  are defined as

$$\text{HDD}_t = \max\{\hat{T} - T_t, 0\}, \quad (1)$$

$$\text{CDD}_t = \max\{T_t - \hat{T}, 0\}, \quad (2)$$

where  $\hat{T}$  is the basis temperature which is determined beforehand. The accumulated heating degree days HDDs and the accumulated cooling degree days CDDs over an  $N$ -day period ending at date  $t$  are defined as

$$X_t = \begin{cases} \sum_{i=1}^N \text{HDD}_{t-i+1}, \\ \sum_{i=1}^N \text{CDD}_{t-i+1}. \end{cases} \quad (3)$$

In the following, we use these HDDs and CDDs are taken as the underlying indices on which the temperature options are written. Moreover, throughout this paper, we pick up the options of European type as a typical temperature derivative. We denote them by

$$B(X_T) = \begin{cases} A \cdot \max\{X_T - K, 0\} & \text{if } B \text{ is call option,} \\ A \cdot \max\{K - X_T, 0\} & \text{if } B \text{ is put option,} \end{cases} \quad (4)$$

where  $K$  is the strike index and  $A$  is the nominal pay-off rate.

Now, we proceed to Davis’s pricing model. As mentioned in Section 1, temperature derivatives are written on non-tradable assets, and hence they can not be priced by using the replicating portfolio. In consideration of this fact, Davis [3] approached to this problem by using marginal substitution value which was proposed as a pricing method for the derivatives in incomplete markets (Davis [2]).

The outline of the pricing method is as follows. An investor with concave utility function  $U$  and an initial cash endowment  $x$  forms a dynamic portfolio whose cash value at time  $t$  is given by  $H_\eta^\pi(t)$  on trading strategy  $\pi \in \tau$ , where  $\tau$  denotes the set of admissible trading strategies. By maximizing the value function  $V(x) = \sup_{\pi \in \tau} E[U(X_T^\pi(T))]$ , the fair price  $\hat{p}$  of the European option  $B(X_T)$  is derived as described below.

Basic Pricing Formula (Davis [2]) :

$$\hat{p} = \frac{E[U'(H_\eta^*(T))B(X_T)]}{V'(\eta)}, \quad (5)$$

where  $V'(\eta) = \frac{d}{d\eta} E[U(H_\eta^*(T))]$ , and  $H_\eta^*(T)$  denotes the optimal portfolio value. On the basis of this formula, Davis [3] gave a fair price of a temperature option on HDDs or CDDs for an energy company in the framework of his financial model under the following assumptions.

Assumption 1 : The underlying index  $X_t$  and the power spot price  $S_t$  are governed by the following geometric Brownian motions

$$dX_t = \nu X_t dt + \gamma X_t dw_1(t), \quad (6)$$

$$dS_t = \mu S_t dt + \sigma S_t dw_2(t), \quad (7)$$

where  $dw_1(t)$  and  $dw_2(t)$  are standard Brownian motions with correlation  $E[dw_1 dw_2] = \rho dt$ .

Assumption 2 : The profit of  $Y_t$  of the company is formed as

$$Y_t = \alpha X_t S_t, \quad (8)$$

where  $\alpha$  is a constant. We note that from Assumptions 1 and 2,  $Y_t$  is also governed by a geometric Brownian motion

$$dY_t = \theta Y_t dt + \xi Y_t dw_t, \quad (9)$$

where

$$\begin{aligned} \theta &= \nu + \mu + \rho\gamma\sigma, \\ \xi &= \sqrt{\gamma^2 + \sigma^2 + 2\rho\gamma\sigma}. \end{aligned}$$

Assumption 3 : The optimal portfolio value  $H_\eta^*$  is equal to  $Y_t$ . That is,

$$H_\eta^*(T) = Y_T, \quad (10)$$

$$\eta = Y_0. \quad (11)$$

For a power generator, this assumption is natural, because it simply generates up to the level of the current demand and sells at the market price.

Assumption 4 : Utility is of logarithmic CRRA type,

$$U(Y_t) = \log Y_t. \quad (12)$$

Then, the fair pricing formula (5) takes the following form :

$$\hat{p} = E \left[ \frac{Y_0}{Y_T} B(X_T) \right]. \quad (13)$$

Moreover, using Gisanov's Theorem, we can rewrite (13) for the option (4) as a formula similar to the Black-Scholes one. The price  $\hat{p}$  is provided by

$$\hat{p} = \begin{cases} Ae^{-qT} \{X_0 \Phi(d_1) - Ke^{-(r-q)T} \Phi(d_2)\} & \text{if } B \text{ is a call option,} \\ Ae^{-qT} \{-X_0 \Phi(-d_1) + Ke^{-(r-q)T} \Phi(-d_2)\} & \text{if } B \text{ is a put option,} \end{cases} \quad (14)$$

where

$$\begin{aligned} r &= \mu + \nu - \gamma^2 - \sigma^2 - \rho\sigma\gamma, \\ q &= \mu - \sigma^2, \\ d_1 &= \frac{\log\left(\frac{X_0}{K}\right) + \left(r - q + \frac{\gamma^2}{2}\right)T}{\gamma\sqrt{T}}, \\ d_2 &= d_1 - \gamma\sqrt{T}, \\ \Phi(x) &= \int_{-\infty}^x \left( \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} \right) dy. \end{aligned}$$

This is the final form of the pricing formula proved by Davis [3].

Now, we proceed to extend this result. That is, we relax Assumption 4 and find a pricing formula for an energy company with a utility function of HARA class under Assumptions 1-3. As mentioned in Section 1, it is difficult to derive option prices explicitly for HARA utility. On account of this, we give our option pricing formula in a form suitable for numerical simulation. In view of Assumption 3, we first rewrite (5) as

$$\hat{p} = \frac{E \left[ \frac{d}{dY_T} U(Y_T) B(X_T) \right]}{\frac{d}{dY_0} E[U(Y_T)]}. \quad (15)$$

Then, we additionally assume that the following equation holds

$$\frac{d}{dy} E[U(Y_T)] = E \left[ \frac{d}{dy} U(Y_T) \right], \quad (16)$$

where  $y = Y_0$ . Since the solution  $Y_T$  of (9) with initial value  $y$  is given by  $Y_T = y \exp((\theta - 1/2\xi^2)T) + \xi w_T$ , the denominator in (15) is rewritten as

$$\begin{aligned} \frac{d}{dy} E[U(Y_T)] &= E \left[ \frac{dU}{dY_T} \frac{dY_T}{dy} \right] \\ &= E \left[ \frac{dU}{dY_T} \left( \frac{Y_T}{y} \right) \right]. \end{aligned} \quad (17)$$

Substituting (17) into (15), we obtain the following expression as the fair price  $\hat{p}$ :

$$\hat{p} = \frac{E \left[ \frac{dU}{dY_T} B(X_T) \right]}{E \left[ \frac{dU}{dY_T} \left( \frac{Y_T}{y} \right) \right]}. \quad (18)$$

This is just the key formula we want. Here we note that this final result has only a “formal” meaning as a formula to carry out numerical simulations since it is not verified that (16) holds exactly.

By using the formula (18), the price  $\hat{p}$  for a utility function of HARA class is derived. A utility function of HARA class is defined as

$$U(Y) = \frac{1}{b-1} (a+bY)^{1-\frac{1}{b}} \quad b \neq 0, 1 \quad (19)$$

on the domain  $\{Y : a+bY > 0\}$  for given constants  $a$  and  $b$ . Note that this function becomes the logarithmic utility function

$$U(Y) = \log(a+Y) \quad (20)$$

for  $b=1$ . In the case of  $a=0$  and  $b=1$ , it reduces to the log CRRA function treated in Davis [3]. This function also becomes the exponential utility function

$$U(Y) = -e^{-\frac{Y}{a}} \quad (21)$$

if  $b=0$  and  $a>0$  (see Ingersoll [7]). Here we note that if  $a \geq 0$ , it is easy to see that (16) holds exactly. Substituting (19)–(21) into (18), we find the price  $\hat{p}$  for these utility functions. The results are shown in Table 1.

Then, as mentioned, we see that each result has a simple form that allows easy calculation of  $\hat{p}$  by Monte Carlo simulation. Furthermore, a risk aversion parameter of the HARA utility functions is a monotonically decreasing function of  $a$  for a fixed  $b$  employed. This fact allows

Utility function	Option price
$U(Y) = \frac{1}{b-1} (a+bY)^{1-\frac{1}{b}}$	$\frac{E[(a+bY_T)^{-\frac{1}{b}} B(X_T)]}{E\left[(a+bY_T)^{-\frac{1}{b}} \left(\frac{Y_T}{y}\right)\right]}$
$U(Y) = \log(a+Y)$	$\frac{E\left[\frac{1}{a+Y_T} B(X_T)\right]}{E\left[\frac{1}{a+Y_T} \left(\frac{Y_T}{y}\right)\right]}$
$U(Y) = -e^{-\frac{Y}{a}}$	$\frac{E\left[e^{-\frac{1}{a}Y_T} B(X_T)\right]}{E\left[e^{-\frac{1}{a}Y_T} \left(\frac{Y_T}{y}\right)\right]}$

Table 1 : Option prices for utility functions of HARA class.

us to perform a sensitivity analysis of an option price with respect to a risk aversion parameter.

### 3 An Air Temperature Model and the Estimation Problem

In this section, we estimate a time series model to describe daily air temperatures by using the empirical data of Nagoya in a similar manner to that in Davis [3]. As in the preceding section, the indices HDDs and CDDs are modeled by a geometric Brownian motion, and hence the distributions are assumed to be log-normal. In consideration of this, as an illustrative example, we also derive a distribution of CDDs derived from the numerical results using the temperature model and examine whether the obtained distribution is close to a log-normal one.

We first give some additional notations and assumptions on daily temperatures  $T_t$ . Suppose that  $T_t$  decomposes into a trend component and a stochastic component as

$$T_t = \bar{T}_t + D_t, \quad (22)$$

where  $\bar{T}_t$  denotes the long-term average temperature on date  $t$  (for instance, if a data period is 16 years,  $\bar{T}_t$  is the simple arithmetic mean for 16 years up to date  $t$ ) and  $D_t = T_t - \bar{T}_t$  denotes the deviation from  $\bar{T}_t$ .

In Davis [3], the stochastic component  $D_t$  was described by a low-order time series (autoregressive) model AR(1). In this paper, we assume that it is governed by the following (autoregressive moving-average) ARMA( $p, q$ ) model

$$D_t = \sum_{i=1}^p \phi_i D_{t-i} + \sum_{i=0}^q \psi_i \epsilon_t, \quad (23)$$

where  $\phi_0 = 1$ , and  $\epsilon_t$  are the standard Gaussian noises (see Hamilton [4]). As observed later, the shape of the autocorrelation function and the partial autocorrelation function of  $D_t$  with respect to the daily temperatures of Nagoya suggest that  $D_t$  should be described by a higher-order ARMA model.

Now, we proceed to estimate the order and parameters of this model on the basis of the empirical temperature data. For this purpose, we use the daily air temperature data in Nagoya for the 16 years period 1990–2005 from the Japan Meteorological Agency’s website (<http://www.jma.go.jp/jma/indexe.html>). Figure 1 shows the shape of the autocorrelation function and the partial autocorrelation function of  $D_t$  for the daily temperatures data, and the results indicate that  $D_t$  is governed by an ARMA model.

To choose the optimal ARMA order, we use the AIC model order selection criteria and

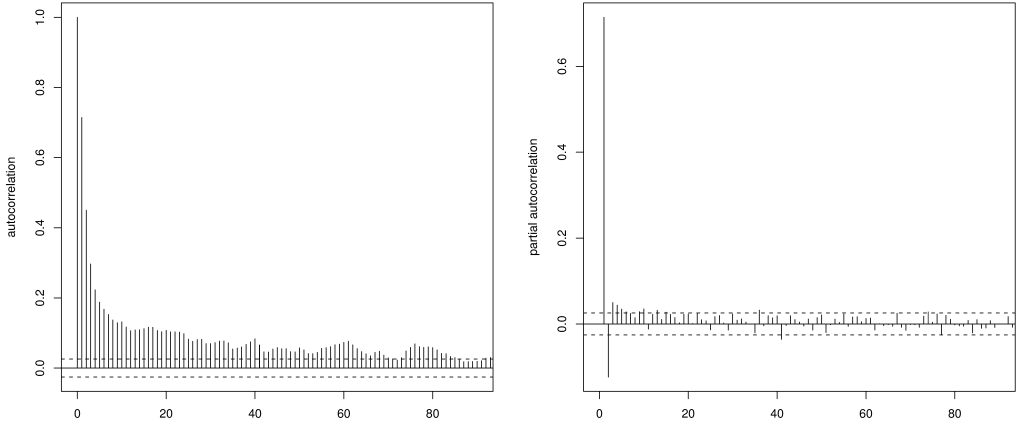


Fig. 1 : The Shapes of autocorrelation and partial autocorrelation functions of the stochastic component  $D_t$  of the empirical data for the daily temperatures in Nagoya.

obtain the following results.

The Estimation results of the Nagoya temperature model :

- The optimal time series model over the 16 years is ARMA(6, 4)
- The estimated values of the parameters are :  
 $\phi = (0.183, 0.273, 0.462, 0.537, -0.652, 0.154),$   
 $\psi = (0.616, 0.0609, -0.493, -0.917),$   
 $\text{Var}(\epsilon) = 2.354.$

where  $\phi$  and  $\psi$  are coefficients vectors defined by  $\phi = (\phi_1, \dots, \phi_p)$  and  $\psi = (\psi_1, \dots, \psi_q)$ , respectively.  $\text{Var}(\epsilon)$  is an estimated value of the variance of  $\epsilon_t$ .

Next, we proceed to generate numerical samples of the index  $X_t$  given by (6) through a simulation of the air temperature model above and examine their distribution. For HDDs on the model, a similar result is obtained, and hence, for brevity, we only demonstrate the CDDs.

Let  $X_T$  be the CDDs at time  $t=T$ , which is accumulated over  $N=92$  days and set the beginning of the period as July, 1. Moreover, the basis temperature  $\hat{T}$  is set at 65 degree F. We generate 5000 samples of  $X_T$  on the basis of the daily air temperature time series model (23) for 92 days, and compare the distribution of  $\log X_T$  and a normal distribution. It is clear that the distribution of  $\log X_T$  agrees well with a normal distribution ; indeed, the distribution statistically passes the normality test.

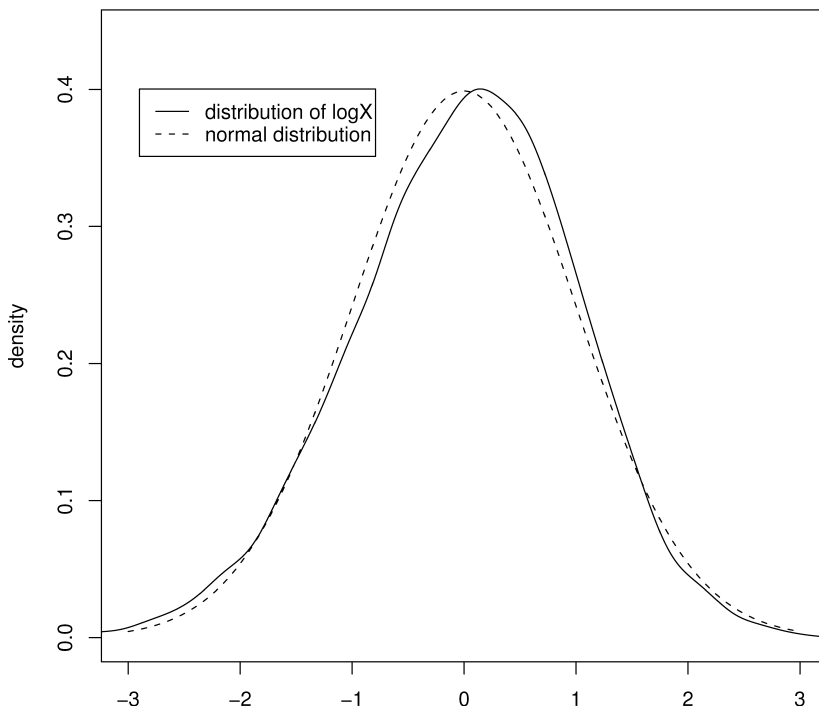


Fig. 2: A Comparison of the density distribution of  $\log X_T$  and normal distribution.

These facts indicate that it is appropriate that the distribution of CDDs is assumed to be log-normal, so that in view of statistical distributions, it may be natural to describe  $X_t$  by a geometric Brownian motion model (6).

*Remark 1:* Figure 3 and 4 show the autocorrelation and partial autocorrelation of the empirical data  $X_t$  and those of the simulated  $X_t$ , respectively. Here the samples start accumulating on 2005/7/1 and cover 92 days. These results show that each sample autocorrelation pattern is very high and cyclic indicating that  $X_t$  is not a Markov process. Hence, in view of stochastic processes, modeling the  $X_t$  data by a geometric Brownian motion (6) may not always be appropriate. Nevertheless, if we treat a European type option whose value only depends on the probability distribution of underlying asset at time  $T$ , such a modeling is fully applicable.

The preceding result also gives an estimation method for the unknown parameters of (6) which  $X_t$  satisfies. Finally, we will sum up this method as a set of procedures.

Note that the distribution of  $\log X_T$  on maturity  $T$  is given as

$$\log X_T \sim N\left(\log X_0 + \left(\nu - \frac{1}{2}\gamma^2\right)T, \gamma^2 T\right). \quad (24)$$

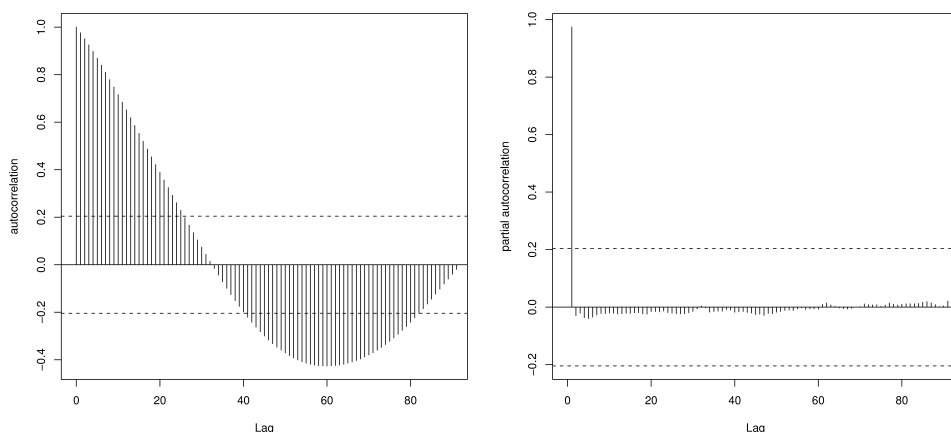


Fig. 3: The shapes of autocorrelation and partial autocorrelation of the empirical data  $X_t$ .

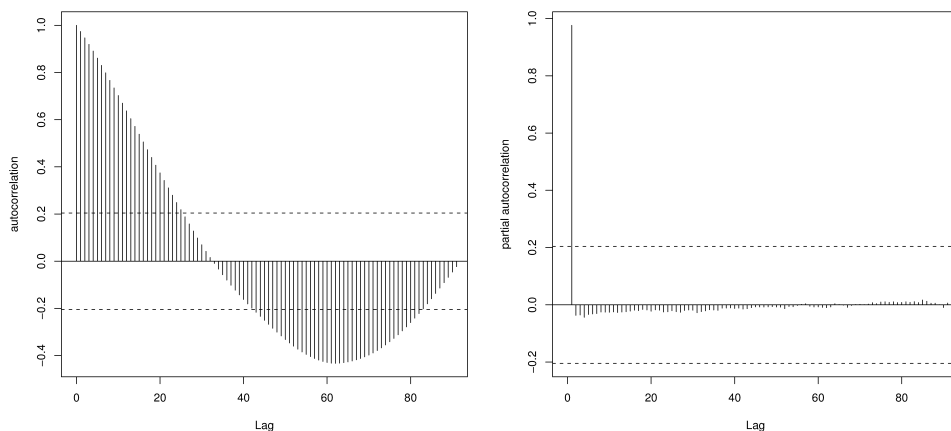


Fig. 4: The shapes of autocorrelation and partial autocorrelation of the simulated  $X_t$ .

If we find out the mean  $m$  and the variance  $s^2$  of  $\log X_t$ , the parameters  $(\nu, \gamma)$  in (6) are obtained

$$\nu = \frac{-\log X_0 + m}{T} + \frac{\gamma^2}{2}, \quad \gamma = \sqrt{\frac{s^2}{T}}, \quad (25)$$

since

$$m = \log X_0 + \left( \nu - \frac{1}{2}\gamma^2 \right) T, \quad s^2 = \gamma^2 T.$$

Therefore, if a time series model of air temperatures fitting empirical data is found, unknown parameters  $(\nu, \gamma)$  can be estimated using following procedures.

(Procedure 1) Simulate  $D_t$  by (23) together with the estimation results and generate the samples.

(Procedure 2) Substitute the samples of  $D_t$  into (22) and generate the samples of  $T_t$ .

(Procedure 3) Accumulate  $T_t$  over  $N$  days and compute  $X_T$ .

(Procedure 4) Repeat (Pro. 1)–(Pro. 3), to generate a large number of samples of  $X_T$ .

(Procedure 5) Calculate a distribution of simulated  $\log X_T$  by (Pro. 4), and estimate the mean  $m$  and the variance  $s^2$  of  $\log X_T$ .

(Procedure 6) Using (25), determine  $\nu$  and  $\gamma$  in (6).

In Table 2, we give some examples of  $(\nu, \gamma)$  estimated by the above method.

$N$	$\nu$	$\gamma$
31	0.0197	0.0232
62	0.0178	0.0130
92	0.0149	0.0105

Table 2: Estimated values of  $(\nu, \gamma)$ .

#### 4 A pricing example on CDDs accumulated on Nagoya summer temperature data

Now, on the basis of the results in the preceding sections, we will calculate call and put option prices on the Nagoya temperature model for an electric power company, and examine their risk sensitivity.

In the followings, we calculate the option prices in the setting below.

- Target securities: European call option and put option on CDDs.
- Maturity: 9/31.
- Basis temperature of the CDDs: 65 degrees F.
- Days over which the CDDs are accumulated: 92.
- Nominal pay-off rate:  $A=1$ . Strike index:  $K=730$ .

In Japan, the electricity market is still not developed sufficiently. Because of this, we assume the price of electricity  $S_t$  is a constant 1. Moreover, for simplicity, we set  $X_0=186.8$  and  $\alpha=1$ .

If the utility function of our approach is of log CRRA type, the option price is determined explicitly by Davis's formula (14). On the other hand, in case of logarithmic, exponential, and

HARA type utility functions, the prices are evaluated using the formula in Table 1 with Monte Carlo simulations.

Then, the option pricing simulation algorithm is as follows.

(Step 1) For given  $\alpha$ ,  $A$ ,  $K$ ,  $S$ , and  $X_0$ , we generate many numerical samples of  $X_T$  estimated by the procedures in Section 3.

(Step 2) Using these samples, we calculate the sample values of  $B(X_T)$  and  $Y_T$ .

(Step 3) Finally, substituting the results of (Step 2) into each formula in Table 1 in Section 2, and calculating the sample mean, we obtain  $\hat{p}$ .

Table 3 shows the explicit solution of the European call and put option values under log CRRA utility given by (14). Note that the parameters  $(\nu, \gamma)$  used in this simulation are estimates shown in Table 2.

The simulation results for the European call and put options under logarithmic, exponential, and HARA utility are shown in Fig. 5-7. Here, to perform risk sensitivity analysis later, in case of logarithmic and exponential utility, we plot the call and put option prices as a function of  $a$ . In case of HARA utility we plot them as a function of  $a$  for fixed values of  $b = -1$ ,  $-1/2$  and  $-1/3$ .

	call option	put option
price	6.99	8.08

Table 3 : The option prices in the case of log CRRA utility .

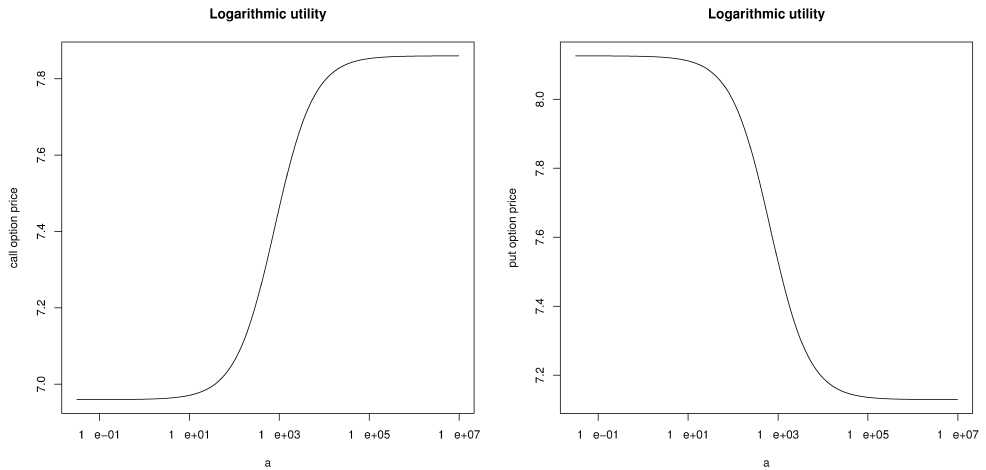


Fig. 5: The option prices for logarithmic utility.

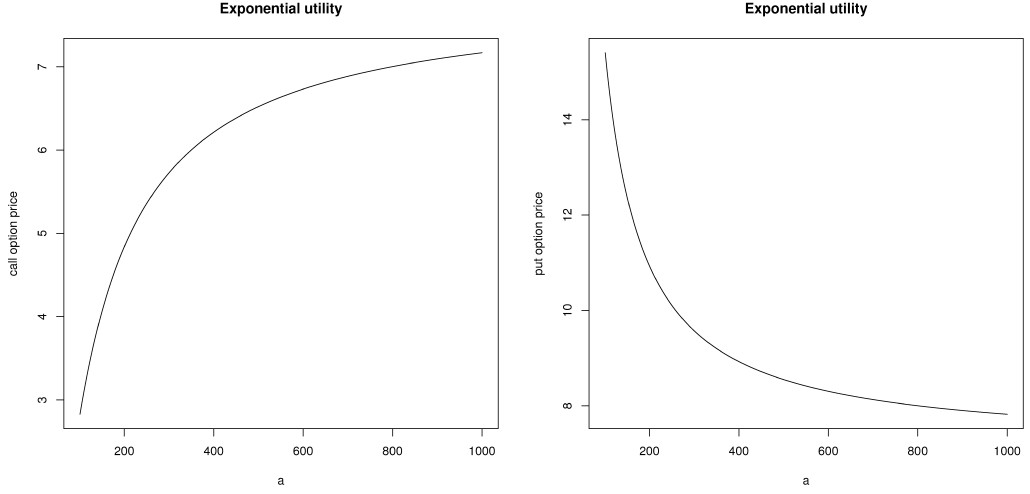


Fig. 6: The option prices for exponential utility.

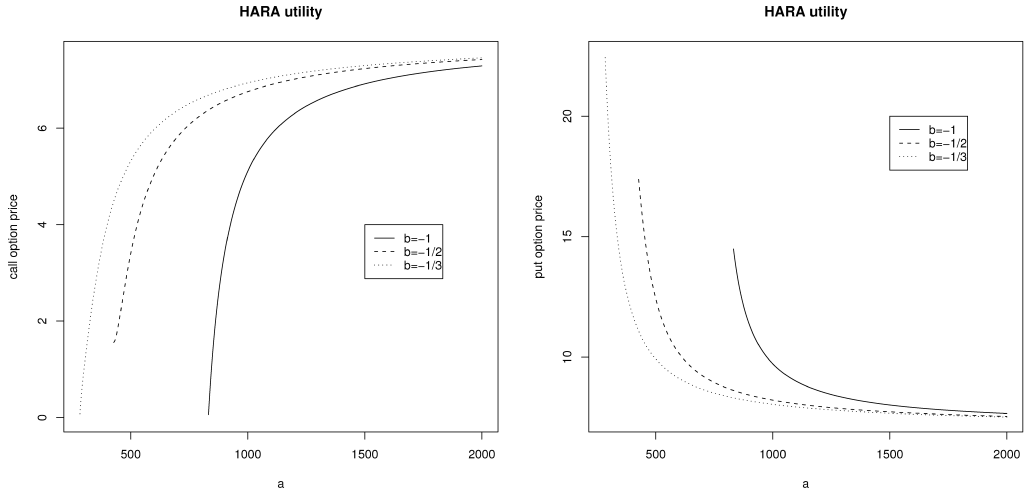


Fig. 7: The option prices for HARA utility.

Now, we examine risk sensitivity of the prices values which is conveniently done with our pricing formula. For such analysis, often ARA (Absolute Risk Aversion) is used, which is well-known as the indicator of the degree of the response to risk, hence we also consider it. In our cases, they are expressed as  $ARA(Y)=1/(a+Y)$ ,  $ARA(Y)=1/a$ ,  $ARA(Y)=1/(a+bY)$  for logarithmic, exponential, and HARA, respectively, since it is defined as  $ARA(Y)=-U''(Y)/U'(Y)$ . These results show that the value of ARA decreases with increasing value of  $a$ . Figure 5 ~7 indicate that the values of the call option prices decrease with increasing value of ARA

for all utility functions. In contrast with this, the values of the put option prices increase in keeping with the value of ARA for all utility functions.

Finally, we proceed to investigate the relation between the put option prices and the ARA. The results on the put options are consistent with the fact that an option purchaser has a taste for hedging temperature risk. Thus, an electric power company purchases the put option to hedge the risk of a battering of its profit by unusually cool summer. Hence, it is natural that the risk-hedging value, i.e. the put option value, for the company increases in keeping with the value of ARA. Thus, the temperature options for an electric company may be regarded as a sort of insurance product.

*Remark 2:* As mentioned in Section 2, HARA utility function is defined on a domain  $\{Y : a + bY > 0\}$ . Therefore, in our numerical calculations, we only use the samples of  $Y_T$  satisfying this condition for a given  $a$  and  $b$ .

## 5 Concluding Remarks

In this paper, we develop Davis's formula for an energy company by extending a class of utility function treated in his work, and derive a pricing formula for HARA utility which is suitable to obtain the value numerically. Then, we estimate a time-series model to describe temperature changes in Nagoya and calculate call and put option prices on CDDs using this pricing formula and Nagoya temperature model.

Furthermore, as a feature of our pricing formula, we examine the risk sensitivity analysis for the calculated price on the basis of ARA, and observe that the movement of the value with respect to parameters is consistent with the movement of ARA ; thus, we see that the price of the temperature option has the property of an insurance product.

We finally offer to some concluding remarks. (1) As shown in Section 3, the assumption that the CDDs data  $X_t$  is described as geometric Brownian motion may not be appropriate, since the empirical data shows the process is not Markovian, while the distribution of  $X_T$  at the terminal time  $T$  as being log-normal seems natural. Therefore, for example, if one treats the options whose process depends on the "paths" of  $X_t$ , such an index should be formulated by better stochastic models. (2) Recently, using an approach similar to that in Davis's work, "utility indifference pricing method" has been developed (e.g. Henderson [5, 6]). This method would allow us to price derivatives in an incomplete market by maximizing an

investor's expected utility through dynamic trading. It would be natural to apply this method to our problem of pricing temperature options and compare the results with those in the present paper. Results of such investigations will be reported in future publications.

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